Directional numeral modifiers: an implicature-based account*

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1. Introduction

Directional numeral modifiers are directional prepositions that can also be used to modify numbers. As shown in (1), the English expression *up to* has these two functions.

(1) a. Mary walked up to the counter.
   b. The store is offering discounts of up to 50%.

Directional numeral modifiers (DNMs) exist in a wide variety of languages. For instance, the Farsi expression *ta* and the Russian *do* can be used as directional prepositions as well as numeral modifiers, as illustrated in (2) and (3).

(2) **Farsi**
   a. Ta labe daryache raftim.
      TA edge of-the-lake went.
      ‘We walked up to the edge of the lake.’
   b. U panj ta shirini khord.
      He five TA cookies ate.
      ‘He ate up to five cookies.’

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Directional numeral modifiers are different from other numeral modifiers in several ways. For example, DNMs do not license NPIs even though they appear to set an upper bound (Schwarz et al. 2012). This can be observed in (4). Here the non-directional numeral modifier at most licenses the NPI ever while the directional numeral modifier up to fails to do so.

(4) a. At most ten cats have ever been to Mars.
    b. *Up to ten cats have ever been to Mars.

As we will see, DNMs differ from other numeral modifiers not only with respect to NPI licensing but also in several other ways: they are not downward monotone, they display the so-called bottom-of-the-scale effect (Schwarz et al. 2012), and they set a cancellable upper bound and a non-cancellable lower bound (Blok 2015).

In this paper I propose an account for these facts which is based on the idea that DNMs assert a lower bound and implicate an upper bound. In addition, I discuss data where DNMs are embedded in several different environments: under negation, with evaluative adverbs, and in questions. These data will lend further support to an implicature-based analysis. While I will use English examples in this paper, my observations are based on a study of fifteen different languages.1 DNMs display the same characteristics in all these languages.

In the next section I will go over the data that demonstrate the differences between directional numeral modifiers and their non-directional counterparts. In section 3 I will discuss the analysis I propose to account for the behaviour of DNMs. I will then analyse the way in which DNMs interact with negation, evaluative adverbs, and question operators in section 4. Section 5 concludes.

2. Directional numeral modifiers are different

In this section, I will discuss some differences between directional and non-directional numeral modifiers. I will first summarise the three differences between up to and at most noticed by Schwarz et al. (2012) and mention to what extent they hold for DNMs crosslinguistically. Then I will go over two more differences first discussed in Blok (2015).

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1The languages I studied are Danish, Dutch, English, Farsi/Persian, French, German, Greek, Hebrew, Hungarian, Italian, Polish, Romanian, Russian, Spanish, and Turkish. I obtained data on these languages by sending out questionnaires and interviewing informants. For all data and judgments I refer the reader to the complete data set, which can be found at http://dominiqueblok.org/work/.
2.1 Schwarz et al.’s observations

Schwarz et al. observe that *up to* behaves differently from other numeral modifiers that set an upper bound, such as *at most*, in three ways. The first contrast is that *up to*, unlike *at most*, does not license NPIs, as mentioned in the introduction of this paper. This is a property that is not specific to *up to* but holds for DNMs across languages. Related to this is the matter of monotonicity. According to Schwarz et al., *up to n* is a non-monotone quantifier. The examples they use are given in (5)-(6).

(5) a. At most three students smoke.
   b. At most three students smoke cigars.

(6) a. Up to three students smoke.
   b. Up to three students smoke cigars.

The authors claim that while (5a) entails (5b), (6a) does not entail (6b). This judgment was not unequivocally confirmed by my informants. While they uniformly agreed that (5a) entails (5b), they had mixed judgments on (6). Some informants thought there was no entailment, others had the intuition that (6a) entails (6b) and yet others felt that the reverse pattern holds; (6b) entails (6a). I will get back to this issue in section 3.

The third difference Schwarz et al. describe is that *up to* is incompatible with the numeral at the bottom of the relevant scale while *at most* is not; the ‘bottom-of-the-scale effect’. For instance, (7a) is infelicitous whereas (8a) is fine.

(7) a. #Up to one person died in the crash.
   b. Up to ten people died in the crash.

(8) a. At most one person died in the crash.
   b. At most ten people died in the crash.

In these examples, the number at the bottom of the scale is the number one because this is the minimal number the predicate can be true of; it is impossible for less than one person to die. In other cases, the bottom-of-the-scale numeral can be higher or lower than one. For instance, if cakes are sold per slice and eggs are sold in cartons of six, the bottom-of-the-scale numeral in (9a) is lower than one (perhaps 1/12) and the bottom-of-the-scale numeral in (9b) is higher than one (it is six). This is why *up to* can be used in (9a) but not in (9b).

(9) a. Jane bought { up to / at most } one whole cake.
   b. Jack bought { #up to / at most } six eggs.
2.2 The bounds of DNMs

As I observed in Blok (2015), DNMs differ from non-directional numeral modifiers in two more ways. First, DNMs set a lower bound that is not defeasible. Second, their upper bound is defeasible.

Let us first turn to the lower bound. (10) directly indicates that \textit{up to} sets a lower bound that cannot be cancelled. The reason for the infelicity of (10b) is that the \textit{if any}-continuation suggests that it is possible that no students will show up, and this contradicts the first part of the sentence. (10a) is felicitous because \textit{at most} does not set a lower bound and is therefore fully compatible with the possibility of no students showing up.

\begin{enumerate}
\item \textbf{a.} At most three students will show up to the lecture, if any.
\item \textbf{b.} ?Up to three students will show up to the lecture, if any.
\end{enumerate}

(11) and (12) provide indirect evidence for the contrast at hand. Here the a-sentences are out because they contain the word \textit{but}. \textit{But} indicates that there is a contrast or antithesis between the two phrases it conjoins. In the a-sentences, there is no such contrast. As \textit{at most} is fully compatible with no-one showing up or the hearer not selecting any presents, the use of \textit{but} is unwarranted. In the b-sentences, on the other hand, \textit{but} can be felicitously used. There is apparently a contrast between up to ten people showing up or choosing ten presents and no-one showing up or not selecting any presents. This indicates that the phrases with \textit{up to} do not include the zero-possibility, and thus that \textit{up to} sets a lower bound above zero.\footnote{I would like to thank an anonymous reviewer of \textit{Console} 2015 for providing me with these examples.}

\begin{enumerate}
\item \textbf{a.} ?I expect to see at most ten people, but maybe no-one will show up.
\item \textbf{b.} I expect to see up to ten people, but maybe no-one will show up.
\end{enumerate}

\begin{enumerate}
\item \textbf{a.} ?You’re allowed to choose at most ten presents, but you can also choose not to select any.
\item \textbf{b.} You’re allowed to choose up to ten presents, but you can also choose not to select any.
\end{enumerate}

These data can be replicated in other languages. Thus, DNMs crosslinguistically set a non-defeasible lower bound.

Let us now consider the upper bound of directional and non-directional numeral modifiers. The b-sentences in (13) and (14) below show that the upper bound of DNMs can be cancelled. Adding ‘perhaps even more’ or ‘or more’ to a sentence with \textit{up to} does not lead to infelicity. As can be seen in the a-sentences, this is not true for sentences with \textit{at most}. Again, this is a property of DNMs across languages rather than an idiosyncrasy of \textit{up to} in English.
(13) a. #At most ten people died in the crash, perhaps even more.
    b. Up to ten people died in the crash, perhaps even more.

(14) a. #Leftovers keep in the refrigerator for at most one week or more.
    b. Leftovers keep in the refrigerator for up to one week or more.

In sum, there are (at least) five ways in which directional numeral modifiers that set an upper bound differ from their non-directional counterparts: 1) DNMs do not license NPIs; 2) DNMs are not clearly downward monotone; 3) DNMs display the bottom-of-the-scale effect; 4) DNMs set a non-defeasible lower bound; and 5) DNMs set a cancellable upper bound. In the next section I show how these contrasts can be accounted for.

3. Two generalisations

3.1 Analysis

The data in the previous section can be accounted for if we assume two generalisations:

1. Directional numeral modifiers set an asserted lower bound and an implicated upper bound

2. All class B numeral modifiers require quantification over a range of values

Let us first turn to the first of these generalisations. The idea that the lower bound set by DNMs is asserted while their upper bound is implicated directly accounts for the data regarding the bounds discussed in the previous section. As asserted semantic content cannot be cancelled but implicatures can, this generalisation correctly predicts that the lower bound set by DNMs cannot be cancelled while the upper bound can.

Furthermore, if we posit that DNMs assert a lower bound but not an upper bound, it follows that they are upward monotone. This is compatible with the observation that they cannot license NPIs. It also explains the monotonicity data. Recall that my informants disagreed on whether DNMs are upward or downward monotone, and generally found the entailment judgments with DNMs extremely difficult compared to the ones with the crosslinguistic counterparts of at most. These blurry intuitions make sense if we assume that DNMs assert a lower bound and implicate an upper bound. While DNMs are upward monotone, the upper bound implicature can cause informants to believe that perhaps DNMs are downward monotone, or the combination of the assertion and the implicature can lead them to believe that DNMs are non-monotone. In short, this account predicts that DNMs are upward entailing and it also predicts that entailment judgments for DNMs are not as clear-cut as the ones for other numeral modifiers. This prediction is borne out.

At this point the reader may be wondering why it is that in language after language, directional prepositions that are used as numeral modifiers assert a lower bound and implicate an upper bound. While I do not have an answer to this question, I would like to point

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Dominique Blok

out that these expressions behave the same way in the spatial and temporal domains. This can be observed in (16). When *up to* is used in a spatial context, as in (16a), the end-point (the spatial equivalent of an upper bound) can also be cancelled. Saying that Harry ran all the way up to his house does not entail that he stopped there. Temporal contexts show the same phenomenon. In (16b), *until* sets a defeasible end-point. If Joan worked until 10pm, this does not necessarily mean that she stopped working at 10pm.

(15)  
   a. Harry ran (from school) all the way up to his house. I think he may even have gone on to run to the football field after that.  
   b. Joan worked (from 9am) until 10pm today. She may have even stayed later than that.

Thus, the prepositions that can function as DNMs appear to simply denote a range of points in space, in time, or on a numerical scale without saying anything about the points past or above the end-point. This way, they leave open the possibility that the relevant predicate also holds of these points.

The second generalisation requires a small detour to some other literature on numeral modifiers. As first observed by Geurts & Nouwen (2007), certain numeral modifiers are incompatible with the speaker having a specific number in mind. Nouwen (2010) calls these numeral modifiers ‘class B’ numeral modifiers. To see the distinction, consider (16).

(16)  
   a. Meryl donated at least €50 to the charity, namely €70.  
   b. Meryl donated more than €50 to the charity, namely €70.

The first half of (16a), with *at least*, suggests that the speaker does not have a specific cardinality in mind, plausibly because she does not know the exact amount of money Meryl donated to the charity. This is why the ‘namely’-continuation is bad in this sentence. In (16b), which contains the numeral modifier *more than*, the continuation is felicitous, which indicates that the first half of this sentence is compatible with the speaker having a specific number in mind. Numeral modifiers like *more than* are referred to as ‘class A’ modifiers. This class also contains *less/fewer than, above, and under*. Numeral modifiers like *at least, at most, minimally, maximally*, and the DNM *up to* are class B modifiers. That *up to* is a class B modifiers is shown in (17). DNMs crosslinguistically belong to class B.

(17) Meryl donated up to €50 to the charity, namely €40.

Now let us return to the second generalisation. The notion that class B modifiers require quantification over a range of values neatly ties in with the observations above: expressions like *at least, at most, and up to* can only be used if the speaker leaves open multiple possible number values. Therefore, a range of numbers to quantify over is in order.

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4In all languages I looked at except English, *up to and until* have the same form. I will therefore assume that *up to* and *until* have the same denotation except for the fact that one is used in spatial contexts and the other is used in temporal contexts.
The range requirement is not only consistent with the data in (16)-(17), it can also account for the bottom-of-the-scale effect in combination with the first generalisation. To see this, consider the data in (18)-(19).

(18) a. #Up to one person died in the crash.
b. Up to two people died in the crash.

(19) a. #At most zero people died in the crash.
b. At most one person died in the crash.

The two generalisations I propose account for these data in the following way. *Up to* sets a lower bound. In the context of the sentences above, this lower bound is one, because this is the minimum number the predicate can be true of (it is not possible for more than zero people but less than one person to die). In (18a) *up to* modifies the number one. For this reason, *up to* quantifies only over the number one. A single number cannot constitute a range of numbers, so (18a) violates the range requirement. The two generalisations thus correctly predict that (18a) is infelicitous. In (18b), on the other hand, there is quantification over the range [1,2]. This is a range, so (19b) does not violate the range requirement and is correctly predicted to be felicitous.

The exact same logic can be applied to the cases in (19). The only difference is that *at most* does not set a lower bound. Therefore *at most* quantifies over only the number zero in (19a). This is a single cardinality and not a range, so (19a) violates the range requirement and is correctly predicted to be bad. In (19b), *at most* quantifies over two numbers: zero and one. [0,1] is a range, so (19b) does not violate the range requirement and is therefore felicitous.

Thus, what we observe here is that the bottom-of-the-scale effect is simply a side-effect of the range requirement. The bottom-of-the-scale effect holds for all class B numeral modifiers that set an upper bound and not just for DNMs. The only reason that the *up to* data differ slightly from the *at most*-data is that the scale of *up to* starts above zero whereas the scale of *at most* starts at zero.

I have shown that we can account for all the data from section 2 if we assume that DNMs assert a lower bound and implicate an upper bound and that all class B quantifiers require quantification over a range of numbers. The first generalisation accounts for the data regarding the bounds of DNMs and their monotonicity and NPI licensing properties. The second generalisation follows naturally from the data regarding the differences between class A and class B modifiers and can account for the bottom-of-the-scale effect together with the first generalisation. In what follows I will show how these generalisations can be formalised.

### 3.2 Formalisation

I formalise these ideas in the degree semantics framework used by Nouwen (2010). The denotation of *up to* I propose is given in (20). Here *up to* stands for all DNMs in the various languages that I studied.
(20) \[ \text{up to} = \lambda n_d \lambda P_{(d,t)} \forall m \in [s,\ldots,n] : P(m) \]
where \( s > 0 \land s \neq n \)

This says that \textit{up to} takes a number \( n \) of type \( d \), the type of degrees, and a degree predicate \( P \) of type \( (d,t) \). It asserts that \( P \) holds for all numbers \( m \) in an interval from \( s \), a contextually given lower bound, to \( n \). For \textit{up to} to set an upper bound, \( s \) must be higher than zero and for the range requirement to hold, \( s \) cannot be equal to \( n \).

I assume with Nouwen (2010) that numeral modifiers combine with the silent counting quantifier \textit{many} given in (21) originally proposed by Hackl (2000).

(21) \[ \text{many} = \lambda n_d \lambda P_{(e,t)} \lambda Q_{(e,t)} . \exists x [ \#x = n \land P(x) \land Q(x)] \]

\textit{Up to} combines with the number it modifies and undergoes QR, leaving a trace of type \( d \). The counting quantifier \textit{many} then takes this trace as an argument. The structure of a sentence like (22) would then be as in (23).

(22) Up to 10 people voted for this party.

(23) \[ \text{up to 10 } [\lambda n \ [n\text{-many people voted for this party} ] ] \]

Assuming a contextually determined lower bound of 1, the denotation of (23) is as in (24).

(24) \[ \forall m \in [1,\ldots,10] : \exists x [\#x = m \land \text{people}(x) \land \text{voted-for-this-party}(x)] \]

This says that for all numbers \( m \) in the range between one and ten, there is a group of people \( x \) with that cardinality that voted for this party. However, this is equivalent to simply stating that ten people voted for the party. For this reason, I follow Nouwen in proposing that the denotation in (24) is blocked by the one in (26), representing the meaning of (25). Nouwen’s reasoning behind this is that a simple form is preferred to a complex form, so when a simple form and a complex form have the same denotation, the simple one blocks the complex one.

(25) Ten people voted for this party.

(26) \[ \exists x [\#x = 10 \land \text{people}(x) \land \text{voted-for-this-party}(x)] \]

The structure is then rescued by inserting a speaker possibility operator, as in (27), yielding the denotation in (28).

(27) \[ \text{up to 10 } [\Box [\lambda n \ [n\text{-many people voted for this party} ] ] ] \]

(28) \[ \forall m \in [1,\ldots,10] : \Box \exists x [\#x = m \land \text{people}(x) \land \text{voted-for-this-party}(x)] \]
Thus, (22) conveys that for all numbers between one and ten, the speaker considers it possible that that many people voted for the party.

Crucially, this denotation does not exclude the possibility that there might be an \(x\) with cardinality 11 or higher of people who voted for the party. The meaning of \textit{up to} is such that it simply says that the predicate holds for a certain range of values and says nothing about the values above it. The upper-bound implicature is a scalar implicature: the speaker chose not to make the utterance that up to 11 or 12 or more people voted for the party, represented in (29). This allows the hearer to derive the implicature in (30): for all numbers \(m\) above 10, the speaker does not consider it possible that the degree predicate \(P\) holds.

\[
\begin{align*}
\text{(29)} & \quad \forall m \in [1, ..., 11] : \lozenge \exists x [\#x = m \land \text{people}(x) \land \text{voted-for-this-party}(x)] \\
\text{b.} & \quad \forall m \in [1, ..., 12] : \lozenge \exists x[\#x = m \land \text{people}(x) \land \text{voted-for-this-party}(x)] \\
\text{c.} & \quad \text{etc.}
\end{align*}
\]

\[
\begin{align*}
\text{(30)} & \quad \forall m > 10 : \neg\lozenge [P(m)]
\end{align*}
\]

In sum, this section shows how two simple generalisations can account for the behaviour of DNMs. I have chosen to formalise these generalisations in Nouwen’s (2010) framework, but I would like to point out that nothing hinges on this particular choice of framework. See Blok (2015) for a formalisation in inquisitive semantics (Ciardelli et al. 2012) along the lines of Coppock & Brochhagen (2013). In the next section I will provide independent evidence for the first generalisation I proposed.

4. Embedded environments

I have claimed that the upper bound of DNMs is implicated rather than asserted. In this section I examine DNMs in various embedded contexts to see if the upper bound behaves like an implicature. In the following two subsections, I will explore how DNMs behave in downward entailing contexts; under negation and in questions. As implicatures generally do not arise in these environments, the prediction is that the upper bound of DNMs should be absent here. This prediction is largely borne out, though I discuss some complications in section 4.2. In section 4.3 I will look at a different kind of context: one where a DNM occurs with an evaluative adverb. These data provide evidence both for the pragmatic nature of the upper bound and for the semantic nature of the lower bound of DNMs.

4.1 Negation

I asked fourteen informants, who speak ten languages between them, to judge sentences where DNMs and the crosslinguistic counterparts of \textit{at most} occur in the scope of negation.\footnote{Specifically, these informants were one speaker of Dutch, one speaker of Farsi, two speakers of French, two speakers of German, one speaker of Italian, one speaker of Polish, one speaker of Romanian, three speakers of Russian, one speaker of Spanish, and one speaker of Turkish.} DNMs generally seem to be infelicitous in the direct scope of negation but better...
when a CP intervenes between negation and the DNM, as can be observed in (31). In other words, they appear to be local PPIs (Spector 2014).

(31)  a. #Annette didn’t buy up to ten books.
     b. (?)I don’t think that Annette bought up to ten books.

My informants saw a clear contrast between the translations of (32a) and (32b) in their languages. By using (32a) the speaker conveys that she thinks that the highest discount will be higher than 70%. (32b), on the other hand, expresses that the speaker thinks that the highest discount will be lower than 70%.

(32)  a. I don’t think there will be discounts of at most 70%.
     b. I don’t think there will be discounts of up to 70%.

The contrast between the sentences in (32) is exactly the contrast that my account predicts. *At most* asserts an upper bound, so when negation takes scope over *at most*, it negates that upper bound. When the upper bound is negated, this means that the highest discount is no longer at or below 70% but higher than 70%, as my informants indicated.

Now let us turn to (32b). Let us assume that the discounts the store offers are numbers that are divisible by ten. Then *there will be discounts of up to 70%* conveys that for all numbers in the range [10,20,30,40,50,60,70], the speaker considers it possible that there will be a discount of that amount. By negating this proposition, the speaker expresses that it is not the case that all numbers in this range are possible discount numbers. This entails that there is at least one number that is not a possible discount number. Let us assume that this number is 70. Then the range of possible numbers is [10,20,30,40,50,60]. The highest possible discount is 60 in this case, and this is lower than 70. Let us assume now that the number that is not a possible discount number is 60. If we assume a monotone semantics of bare numerals, this means that once the predicate does not hold for a certain number \( n \) on the scale, it does not hold for all numbers above \( n \), either. Therefore, by excluding 60 the speaker also excludes 70. Thus, the range of possible numbers is [10,20,30,40,50]. In the same way, if we pick 50 as the number the speaker does not consider possible, the range is [10,20,30,40], if we pick 40 it is [10,20,30], etc. What all these possibilities have in common is that the upper bound is lower than 70. Regardless of which number we pick, they all lead to a range with an upper bound that is lower than 70. Therefore, the meaning of (32b) is that the highest discount the speaker considers possible is lower than 70%.

To illustrate the crosslinguistic aspect of these judgments I give the Italian equivalent of (32) in (33). My Italian informant agreed that using the DNM *fino a* leads to a reading where the highest discount is lower than 70% while the same sentence with *al massimo*, ‘at most’, yields a reading where the highest discount is higher than 70%.

(33)  a. Non credo che ci saranno sconti del 70% al massimo.
     Not I believe that there will be discounts of 70% at most.
     ‘I don’t think there will be discounts of at most 70%.’
Directional numeral modifiers: an implicature-based account

b. Non credo che ci saranno sconti fino al 70%.
   Not I believe that there will be discounts FINO A 70%.
   ‘I don’t think there will be discounts of up to 70%.’

These data provide evidence in favour of an implicature-based theory of DNMs. If DNMs asserted their upper bound like other upper-bounded numeral modifiers such as at most, we would expect no contrasts between the two if we embed them under negation. Clearly we do see a contrast. In what follows I will turn to DNMs in questions.

4.2 Questions

Some of my informants also gave their judgments on DNMs that occur in questions. As it has been widely observed that implicatures disappear in questions, this account predicts that DNMs do not give rise to an upper bound in this environment.

I presented eight speakers of seven languages with the scenarios in (34) and (35).  

(34) Helen ate 12 biscuits.
    John asks: ‘Did Helen eat { at most / up to } 10 biscuits?
    Martha gives one of the following answers:
    a. No, Helen ate 12 biscuits.
    b. Yes, in fact, she ate 12 biscuits.

(35) There are 30 rooms in the castle.
    Sarah asks: ‘Are there { at most / up to } 20 rooms in the castle?’
    Phil gives one of the following answers:
    a. No, there are 30 rooms in the castle.
    b. Yes, in fact, there are 30 rooms in the castle.

I asked the speakers to translate the questions and answers to their native language and to indicate whether the a-answers and b-answers were felicitous. The Farsi (36), with the DNM ta, is an example of a translation that was given.

(36) a. Aja Helen ta dah ta biscuit xord?
    QUESTION OP. Helen TA ten biscuit eat?
    ‘Did Helen eat up to ten biscuits?’
    b. Na, Helen davazdah ta xord.
    No Helen twelve ate.
    ‘No, Helen ate twelve.’

My informants were two speakers of French and one speaker each of Dutch, Farsi, German, Greek, Hebrew, and Romanian.
The idea behind this setup is that if the question is read with an upper bound meaning, only the a-sentences should be felicitous. If there is no upper bound, the b-sentences should be felicitous.

As expected, all informants said that the a-answers were the only felicitous answers in the *at most* versions of these scenarios, which indicates that *at most* sets an upper bound. Interestingly, there was a large contrast between the DNM versions of (34) and (35). There was one speaker who said that both (34a) and (34b) were felicitous answers, two speakers who thought neither were felicitous, and five speakers for whom only (34a) was a felicitous answer. This means five out of eight informants got an upper bound reading for *up to* in a the context of a question.

For (35), by contrast, only one speaker said that (35a) was the only felicitous answer, and one speaker said that neither answer seemed right. All other speakers considered (35b) a possible answer. Out of these, three speakers felt that (35b) was the only possible felicitous answer to the question and three others said that both (35a) and (35b) were possible.

The contrast between (34) and (35) may be due to the fact that when you ask about the number of rooms in a castle, presumably this is because you need space in the castle for some event. If you ask whether there are ‘up to 20’ rooms, this is probably because you need 20 rooms, and the ‘yes’-answer indicates that the castle has enough space for you. In other words, you’re reasoning about a minimum number, and if 20 is the minimum, 30 is also acceptable. In (34) there is no obvious contextual minimum, so my informants may have interpreted the numbers as precise numbers rather than minima.

A more important question, though, is why my informants were able to get a reading with an upper bound at all. If the upper bound is an implicature, this is unexpected. I will outline two possible explanations for this fact.

The first way to account for the occurrence of the upper bound in questions is to posit the existence of local implicatures, as in e.g. Chierchia et al. 2009 and Spector 2004. The authors of these papers show that it is possible for implicatures to survive in downward entailing environments. In (37), for example, *some* carries a ‘not all’-implicature even though it occurs in the scope of negation. If it didn’t, the second part of the utterance would contradict the first part, and this is not the case.

(37) I don’t expect that some students will do well, I expect that all students will do well.

In a similar vein, DNMs may give rise to local implicatures depending on the context, and this could explain the contrast we observe between (34) and (35) without giving up on the idea that the upper bound of DNMs is implicated.

Another explanation could be that the implicature DNMs give rise to is in the process of fossilisation (Aloni et al. 2016). This may mean that it is in some intermediate position between being a full implicature and being part of the asserted content. This would explain
the observed speaker variation for the judgments of (34) and (35). Some speakers or languages may not have fossilised the implicature or may be in an early stage of fossilisation, while other speakers or languages are further ahead in the process of fossilisation.

We have seen that while DNMs behave exactly as one would expect in the scope of negation, their behaviour in questions is unexpected given the pragmatic approach I have proposed. This may be due to the existence of local implicatures, a process of fossilisation of the implicature, or some other factor. I will leave this as an open issue. What is clear is that DNMs can be interpreted without an upper bound in questions, and this is not the case for numeral modifiers like at most. Therefore, the fact that there is a contrast between the upper bound of these two kinds of expressions remains uncontested.

### 4.3 Evaluative adverbs

Aside from operators that create a downward entailing environment, evaluative adverbs such as fortunately are another good way to test implicatures. As observed by Nouwen (2006), evaluative adverbs target the assertion of a sentence and not its implicature. Consider the examples in (38).

\[
(38) \quad \text{a. Fortunately, some students attended the wedding.} \\
\text{b. Fortunately, the soup is warm.}
\]

In (38a) the speaker is expressing his joy about the fact that there were students who attended his wedding, not about not all students attending his wedding. Likewise, in (38b) the speaker conveys that she is happy that the soup has a certain minimum temperature; she is not saying that she is happy that the soup is not hot.

With these tools in hand, let us turn to DNMs. There is a clear contrast between (39a) and (39b).

\[
(39) \quad \text{a. Fortunately, at most 100 people will attend my wedding.} \\
\text{b. Fortunately, up to 100 people will attend my wedding.}
\]

When fortunately occurs with at most, it targets the upper bound: the speaker is happy that no more than 100 people will attend the wedding. When it occurs with up to, the sentence ends up conveying joy about the high number of guests. In other words, the speaker is happy about the high number of guests he is expecting, and not about the upper bound of 100. These facts indicate that the asserted content of at most is the upper bound, while the asserted content of up to is the lower bound. The data with evaluative adverbs do not only shed light on the upper bound implicature of DNMs but also demonstrate once again that the lower bound is part of the semantic content of DNMs. These data, too, have been confirmed by speakers of different languages.\(^7\)

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\(^7\)In this case, the judgments came from the same set of informants as the judgments in sections 1-3 and are thus about the same fifteen languages (see footnote 1).
5. Conclusion

I have proposed that there is a crosslinguistic category of directional numeral modifiers that behave differently from other numeral modifiers that set an upper bound. I have argued that they assert a lower bound and only implicate an upper bound. This proposal along with the idea that all class B numeral modifiers are subject to a range requirement accounts for a number of differences between DNMs and other upper-bounded numeral modifiers. Finally, I discussed data where DNMs occur under negation, in questions, and with evaluative adverbs. These data confirm my proposal regarding the nature of the bounds of DNMs, although more research is needed on the way DNMs behave in questions.

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References